Optimal in-store inventory policy for omnichannel retailers in franchising networks

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Abstract

Purpose – The purpose of this paper is to characterize the optimal ordering and allocation policy for a store replenishment decision in the context of an omnichannel retailer in a franchise network. The authors further show that a myopic policy is optimal, which circumvents the curse of dimensionality for the multi-period inventory model and help store managers optimize their decisions about the amount of inventory to stock for both online and offline demands and the percentage of inventory to reserve for online orders.

Design/methodology/approach – This research is trigged by several managerial studies which suggest reserving a certain percentage of the in-store inventory for online orders as a good store inventory allocation practice for omnichannel retailers in a franchise network. The authors used an analytical model to develop this practice by clarifying how store managers can decide on the amount of inventory to replenish and the percentage to reserve for online orders.

Findings – This study develops a finite horizon, periodic review inventory model to identify an optimal and dynamic replenishment and allocation policy. The analysis uncovers the system's fundamental structural property concavity. The research shows that, due to this property, the optimal replenishment policy is a base-stock policy. The latter is due to the base stock level being independent of the initial inventory at hand, and the optimal allocation level being non-decreasing on the base-stock level.

Research limitations/implications – This study contributes to the literature on store inventory management for omnichannel retailers in a franchise network by investigating their optimal store inventory ordering and allocation policy. Nevertheless, the zero-lead time and zero-setup cost assumptions limit the findings.

Practical implications – Insights into an optimal store inventory policy may guide franchisee store managers to decide on the amount of inventory to replenish and the percentage to reserve for online orders.

Originality/value – The originality of this paper lies in its focus on in-store inventory management for omnichannel retailers in a franchise network. The findings are helpful for franchisor retailers to implement the omnichannel strategy at the level of in-store inventory management. Beyond using incentive systems, the franchisor should leverage legitimate powers by mentioning a relevant measure in their contracts with their franchisee to minimize their channel conflicts and ensure their customers have seamless shopping experiences.

Keywords Omnichannel retailing, Analytical model, Inventory optimization, Store fulfilment

Paper type Research paper

Introduction

The proliferation of sales channels and ongoing digitalization has changed the retail landscape dramatically in recent years (Verhoef *et al.*, 2015). Retailers' search for synergy in their multiple channels and their goal to provide their clients with a seamless shopping experience motivates them to move from a multi-channel to an omnichannel retailing model (Brynjolfsson *et al.*, 2013; Rigby, 2011). Hansen and Siew Kien (2015) define omnichannel retailing as "an integrated multichannel approach to sales and marketing." Verhoef *et al.* (2015, p. 176) develop this approach as "the synergetic management of the numerous

available channels and customer touch points, in such a way that the customer experience across channels and the performance over channels are optimized." The adoption of an omnichannel strategy creates opportunities for retailers to optimize their firm-level performance by leveraging the assets of one channel for other channels and by coordinating their operations across channels (Cao and Li, 2015). For example, store-based retailers may use their in-store inventory to fulfill orders from other channels to gain several advantages. The advantages are, on the firm side, that store fulfillment generates store traffic, providing retailers with opportunities to cross-sell and up-sell (Neslin *et al.*, 2006) and to form a relationship with their customer base, which they cannot do when sending parcels. In addition, this strategy allows retailers to use their in-store inventory efficiently and reduces the need for in-store markdowns (Elnaz *et al.*, 2015). On the customer side, store fulfillment provides customers with the freedom to use different channels in different situations (e.g. buying online but picking goods up in-store) (Berry *et al.*, 2010). This strategy also allows online and mobile consumers to save postage costs and to receive orders sooner than when an order is sent from a centralized distribution center (DC) (Elnaz *et al.*, 2015).

Store fulfillment may also produce different channel conflicts for omnichannel retailers, such as cannibalization (merely shifting sales from one channel to another), differences in prices and margins across channels (Zhang *et al.*, 2010), and difficulties of inventory allocation between channels if other channels have a strong demand for the limited in-store inventory (Chen *et al.*, 2011). Redesigning incentive systems is a common practice for omnichannel retailers to encourage coordination and collaboration between channels (Cao, 2014; Steinfield *et al.*, 2002; Payne and Frow, 2004). For example, online and offline channels can share the revenue generated by an order bought online but picked-up in-store. Although such practice is relevant to any organization, its effectiveness varies in terms of retailers' organizational structure (company-owned chains vs franchise network) (Wiener *et al.*, 2018). For company-owned chains, both online and offline channels belong to the same company. Therefore, the design of the incentive system is relatively easy. However, it is difficult to design such incentive systems in a franchise network because it involves benefit allocations between two different entities (franchisor vs franchisee). The franchisor tries to optimize its benefits, whereas the franchisee strives to maximize its own benefits within its territory (Cliquet and Voropanova, 2016; Nair *et al.*, 2009). Despite the existence of incentive systems, store associates may have little motivation to fulfill online orders when in-store inventories are limited. Store-level managers should make the decisions about selling products to online customers at a lower net revenue, as the store only keeps a part of the sales revenue and may prefer to reserve products for a later sale at full revenue to its in-store customers (Chen *et al.*, 2011). Furthermore, store managers should also make the trade-off between serving online customers and ensuring an adequate service level to its own in-store customers if the service capacity is limited (Berry *et al.*, 2010). To address these issues, retailers can leverage other types of power (e.g. coercive, expert, referent and legitimate power) rather than offer rewards (incentive system) to motivate their physical stores to serve online customers (Raven and French, 1958). Although all of these types of power are relevant to franchising arrangements, the degree of power depends on the extent to which the franchisees depend on the franchisor in the relationship (Dapiran and Hogarth-Scott, 2003). For example, the franchisor can impose its franchisees to serve online customers only under certain conditions, such as, when there is a strong brand name, an efficient marketing strategy, well-protected intellectual property or other resources that are valued by the franchisees (Frazer *et al.*, 2007). Therefore, omnichannel retailers in franchise networks encounter many more challenges than those with company-owned chains in implementing store fulfillment strategies.

The failure to implement store fulfillment strategies often leads to inconsistent customer experiences (Zhang *et al.*, 2010). Despite the specific challenges for a franchise-based retailer serving its online customers through its franchisees' in-store inventory, the franchisor has to

find operational solutions that reduce the conflicts between physical and digital channels because it should satisfy customers' need for a seamless shopping experience across channels (Lemon and Verhoef, 2016). To reduce the tensions between franchisor and franchisee, the franchisor may consider the optimization of its inventory management not only from its own perspective (firm-level inventory management), but also from its franchisees' perspective (in-store inventory management). Therefore, the optimization of in-store inventory for responding to the demands from both online and offline channels becomes an important topic for omnichannel retailers in franchise networks.

Despite increased attention paid to omnichannel retailers' inventory management, theoretical and empirical knowledge of this topic remains limited and offers very few insights to help store managers in franchise networks make optimal store replenishment decisions. Three specific problems make the prior literature inadequate. First, most studies on in-store inventory management in a multichannel context are based on a store-centric perspective (e.g. Chen *et al.*, 2011; Elnaz *et al.*, 2015). These studies' analytical models only describe the optimization of in-store inventory in response to in-store and online customer demand in terms of the maximization of in-store profits. Customers' seamless shopping experiences are therefore not ensured because the models allow offline stores to refuse online orders for an economic benefit. Second, an increasing number of studies have attempted to address the issues of channel conflicts (Cheng *et al.*, 2016; Xu *et al.*, 2017). Their analytical models describe optimizing firm-level inventory ordering and allocation through online and offline orders, rather than focusing on store-level inventory management. However, the study of inventory optimization at the store level is particularly important for omnichannel retailers in franchise networks, as previously argued. Third, a recent study (Choi *et al.*, 2017) specifically addresses online–offline channel conflicts regarding franchised fashion stores' inventory management. They suggest optimal choices for ordering arrangements between online and offline channels, focusing on the optimal ordering time and the best franchising contract to choose for the fashion brand, instead of exploring optimal store inventory ordering and allocation policy between online-offline. Furthermore, their suggestions are based on an important assumption that the franchisor will first supply the product to the franchisee to sell offline in the first period; after that, the franchisor will sell the product online directly in the second period. Consequently, only certain retailers, if their products are categorized as "fashion products," can adopt the optimal policies proposed by this study to avoid channel conflicts and cannibalization between the franchisee and the brand owner (franchisor). Otherwise, postponing the introduction of online ordering does not make sense. Therefore, retailers selling non-fashion products need to find a different solution.

Managerial studies (ENC, 2016; Hobkirk, 2016) suggest certain store inventory allocation practices regarding online and offline orders, such as the "earmarking" of inventory by channel (i.e. reserving a certain percentage of the in-store inventory for orders from other channels). Franchisors can thus require franchisees to allocate their inventories to either an online or offline channel by stipulating the associated terms and measures in their franchise contracts. Beyond leveraging reward power (incentive mechanisms), implementing the above practice through legitimate power sounds especially promising for omnichannel retailers in a franchise network if the franchisors' resources allow them to do it. Such a measure can optimize in-store inventory management whilst addressing the problem of channel conflicts. Therefore, our study's objective is to identify an optimal and dynamic ordering and allocation policy that could quickly calculate the base-stock level and the allocation level of online and offline channels for store-level replenishment decisions.

An optimal store inventory policy should guide the store managers of omnichannel retailers in a franchise network on how to decide on the amount of inventory to replenish and the percentage to reserve for online orders. We therefore endeavored to specifically develop a finite horizon, periodic review inventory model and identify an optimal and dynamic

replenishment and allocation policy. We uncovered a fundamental structural property of the system – $L^{\text{è}}$-concavity. We used this property to show that a base-stock policy, where the base-stock level is independent of the initial inventory at hand, is the optimal replenishment policy, and the optimal allocation level is non-decreasing on the base-stock level.

The paper is organized as follows: the next section describes the analytical model, followed by the optimal policy. We then show that the myopic policy is optimal. A numerical study is further conducted to provide additional insights. The final section discusses future research directions.

Analytical model

We consider the store-level inventory replenishment and allocation control of a single product for an omnichannel retailer in a franchise network. The franchise contract stipulates that the inventory for any franchised store should respond to two types of demands: those from the clients who buy in-store (offline channel) and pick up the product in-store from the franchisee; and those from the clients who buy online from the franchisor and pick-up the product in-store from the franchisee. The franchisee replenishes and allocates the in-store inventory for customers who buy through either online or offline channels. Once the inventory has been allocated, it is not shared across channels, i.e., if there is sufficient inventory for one channel, consumers from the other channel cannot access its inventory (Hobkirk, 2016). We study a periodic review system with a finite horizon of length T. At the beginning of each period, the retailer needs to make a replenishment decision and an allocation decision. We assume that the franchisor charges the same price for the same product whether bought offline or online, as most multichannel retailers tend to set the same price across channels to avoid their customers perceiving an inconsistency and being dissatisfied (Cao and Li, 2015; Davis *et al.*, 2000; Schramm-Klein and Morschett, 2006; Gölgeci *et al.*, 2018). The selling price is denoted by p, and the unit purchase cost by c. Let h be the unit holding cost per period. To make the sales profitable, we assume that $p \geqslant c$. We further assume that the ordering lead time for the franchisee is negligible. We argue this assumption from the perspective of power-based behavior in supply chain management. Gölgeci *et al.* (2018) suggest that the quick response to a franchisee's requirement to replenish stock is an important source of the franchisor's power *vis-à-vis* its franchisee in supply chains. Given that the franchisor's power in the franchisor–franchisee relationship is critical to solving channel conflicts, the franchisor is motivated to reduce the franchisee's ordering lead time by optimizing its local DC and/or adopting new technologies. Thus, the ordering lead time can be very short and was even negligible in our study. From the same perspective of power-based behavior in supply chain management, we set our study in the scenario where, at the beginning of each period, the franchisor sends out a vehicle full of enough inventory to make a tour and stop at each retail store. Therefore, we assume there is no fixed ordering cost. Furthermore, the franchisor provides incentives to encourage the franchisee to use his stock to satisfy a portion of the online customers (Steinfield *et al.*, 2002; Cao, 2014; Payne and Frow, 2004). Given the revenue-sharing policy is used as an effective practice to solve channel conflicts (Koulamas, 2006; Zhao *et al.*, 2016), we assume that the franchisor pays the franchisees a revenue share equal to r ($r < p$) for each unit sold in order to incentive the franchisees to serve customers who buy online but pick-up their orders in-store. In a generic period t, $1 \leqslant t \leqslant T$, offline and online demands, denoted, respectively, by D_1 and D_2, are stochastic and independent of each other. They follow known distributions $F_1(\cdot)$ and $F_2(\cdot)$ with means of μ_1 and μ_2, respectively. However, it is worth noting that our results can be easily extended to cases where offline and online demands are correlated.

At the beginning of each period, the franchisee examines the inventory on hand and makes a replenishment decision. The order will arrive immediately. Thereafter, a decision is made on how much stock to allocate to each channel. Once the decision has been made, the stock allocated to one channel cannot be used to meet demands from the other channel. After replenishment and allocation, demands from different channels are

realized. The on-hand inventory allocated to each channel is used to satisfy its channel's demand. If the online demand cannot be fulfilled, the DC will switch to another retail store, meaning that the first retailer will lose the demand. However, if the offline demand cannot be fulfilled due to a stock-out, we assume that it will be backlogged with a backordering cost b per period. If, at the end of one period, there is remaining inventory allocated to one channel, we assume that this inventory, together with the new order, can be reallocated in the next period. Our objective is to investigate the optimal replenishment and allocation policies that maximize the expected total discounted profit during the time horizon.

The optimization problem can be formulated as a Markov decision process. Let x and y be the inventory levels before and after an order is placed in a generic period t. Let z be the quantity of inventory allocated to the offline demand; thus, $y-z$ is the quantity allocated to the online demand. The function $f_t(x)$ represents the expected total discounted profit in period t, given that the inventory level at the beginning of the period is x. Note that x may be negative when unmet offline demand is backordered. It is also easy to verify that a negative inventory level will never be profitable. We can therefore assume that the optimal inventory level after ordering at the beginning of a period is always positive.

The dynamic program for $t = 1, \ldots, T$ can be written as follows:

$$f_t(x) = \max_{y \geqslant x}\left\{-c(y-x) + \max_{0 \leqslant z \leqslant y} E\left\{(r+p)(y-z) - (r+p)(y-z-D_2)^+\right.\right.$$
$$-h(y-z-D_2)^+ + pz - p(z-D_1)^+ - h(z-D_1)^+ - b(D_1-z)^+$$
$$\left.\left.+\alpha f_{t+1}\left((y-z-D_2)^+ + z-D_1\right)\right\}\right\}. \tag{1}$$

The two domain constraints indicate that the inventory level after replenishment is at least as much as that before replenishment and that the allocation to one channel cannot exceed the total inventory. The terms in Equation (1) include the variable ordering cost, revenue from online and offline demand, inventory-related costs and expected discounted future profits. We can rewrite Equation (1) as follows:

$$f_t(x) = \max_{y \geqslant x}\left\{-cy + G_t(y)\right\} + cx, \tag{2}$$

where:

$$G_t(y) = \max_{0 \leqslant z \leqslant y} E\left[-rz - (r+p+h)(y-z-D_2)^+ - (p+h)(z-D_1)^+ - b(D_1-z)^+\right.$$
$$\left.+\alpha f_{t+1}\left((y-z-D_2)^+ + z-D_1\right)\right] + (r+p)y$$
$$= \max_{0 \leqslant z \leqslant y} J_t(y,z) + (r+p)y, \tag{3}$$

and:

$$J_t(y,z) = E\left[-rz - (r+p+h)(y-z-D_2)^+ - (p+h)(z-D_1)^+ - b(D_1-z)^+\right.$$
$$\left.+\alpha f_{t+1}\left((y-z-D_2)^+ + z-D_1\right)\right].$$

At the end of the time horizon, without loss of generality, we assume that $f_{T+1}(x) = cx$.

Optimal policy
In this section, we investigate an optimal ordering and allocation policy. First, we introduce the concepts concavity, supermodularity and $L^\natural$-concavity.

Given a set C in $\mathfrak{R}^n$, a function $f\colon C \to \mathfrak{R}$ is concave over set C, if for any x, $x' \in C$ and $\lambda \in [0, 1]$, $f(\lambda x + (1-\lambda x') \geqslant \lambda f(x) + (1-\lambda)f(x')$. Suppose X is a subset in $\mathfrak{R}^n$ and a function $f\colon X \to \mathfrak{R}^n$. The function f is supermodular on the set X if for any $x, x' \in X, f(x) + f(x') \leqslant f(x \wedge x') + f(x \vee x')$, whenever $x \wedge x'$, $x \vee x' \in X$, where $x \vee x' = (\max\{x_1, x'_1\}, \ldots, \max\{x_n, x'_n\})$ and $x \wedge x' = (\min\{x_1, x'_1\}, \ldots, \min\{x_n, x'_n\})$. We call a function $f(x)$ $L^{\grave{e}}$-concave if $g(x, z) = f(x - z e_n)$ is supermodular on $X \times \mathfrak{R}$. $L^{\grave{e}}$-concavity implies convexity and supermodularity.

In the next proposition, we show that the value functions have the following structural properties:

    P1. For $t = 1, \ldots, T$, (a) $f_t(x)$ and $G_t(x)$ are concave; (b) $J_t(y, z)$ is $L^{\grave{e}}$-concave in y and z.

These structural properties are critical to characterize the optimal ordering and allocation policy. Specifically, the concavity of $G_t(x)$ ensures that there is a base-stock such that if the initial inventory on-hand is lower than this level, it is optimal to order up to the base-stock level, and if the initial inventory is higher than this level, it is optimal not to order. $L^{\grave{e}}$-concavity in y and z ensures that the optimal allocation level is monotone in terms of the inventory level y. Let y^* be the optimal order-up-to level and $z^*(y)$ be the optimal allocation quantity given the inventory after ordering is y. We present the optimal policy in the next proposition:

    P2. In period $t = 1, \ldots, T$, (a) the optimal ordering policy is a base-stock policy. That is, there is a base-stock level y^* such that if $x < y^*$, it is optimal to order up to y^*; if $x \geqslant y^*$, it is optimal not to order; and (b) $z^*(y) \leqslant z^*(y+\varepsilon) \leqslant z^*(y) + \varepsilon$ for any $\varepsilon \geqslant 0$.

P2 (a) indicates that there is an optimal order-up-to level that is independent of the initial inventory level and (b) shows the monotone property and bounded sensitivity of the optimal allocation quantity in terms of the optimal order-up-to level. This result is intuitive since if the total inventory level is higher, it is desirable to allocate more to each channel.

Myopic policy
P2 indicates that there is a state-independent optimal base-stock level for ordering and an optimal allocation level that is non-decreasing in the inventory on hand. To make the levels easy to implement, it is important to study how to calculate the optimal base-stock level and the allocation level. In this section, we consider the myopic optimization solution of the model. Myopic means looking only at the "current" one-period problem. A policy is said to be myopic if it is the optimal policy for a single period model that is defined explicitly in terms of the original model parameters.

Let $\bar{f}_t(x) = f_t(x) - cx$. Equation (1) can be written as follows:

$$\bar{f}_t(x) = \max_{y \geqslant x} \left\{ (r+p-c)y + \max_{0 \leqslant z \leqslant y} E\left[(\alpha c - r)z - (r+p+h-\alpha c)(y-z-D_2)^+ \right. \right.$$

$$\left. \left. -(p+h)(z-D_1)^+ - b(D_1-z)^+ + \alpha \bar{f}_{t+1}\left((y-z-D_2)^+ + z - D_1\right) \right] \right\} - \alpha c \mu_1,$$

and:

$$\bar{f}_{T+1}(x) = 0.$$

We denote by $z^*(y)$ the optimal allocation solution for:

$$\theta(y) = \max_{0 \leqslant z \leqslant y} E\left[(\alpha c - r)z - (r+p+h-\alpha c)(y-z-D_2)^+ \right.$$

$$\left. -(p+h)(z-D_1)^+ - b(D_1-z)^+ \right],$$

and y^* as a maximizer of $\pi(x) = \max_{y \geqslant x}\{(r+p-c)y+\theta(y)\}-\alpha c\mu_1$. Following Porteus (2002), we call a state where $x \leqslant y^*$ consistent. Since $\pi(x)$ attains its maximum value at y^*, for any consistent state x, the optimal decision is to order up to y^*, that is, $\pi(x) = (r + p - c)y^* + \theta(y^*) - \alpha c\mu_1 = \pi(y^*)$:

P3. Suppose that $f_{T+1}(x) = cx$. Then, when a state where $x \leqslant y^*$ is observed at the beginning of a period, it is optimal to order up to the quantity y^* in the period and the following periods.

P3 indicates that if the initial inventory level does not exceed the base-stock level y^*, then finding the optimal base-stock level and the optimal allocation quantity is reduced to a single-period optimization problem. From the proof of P3, using the Leibniz rule, it is easy to verify that the optimal allocation quantity has the following property:

Corollary 1. If the terminal value function is given by $f_{T+1}(x) = cx$ and a state $x \leqslant y^*$ is observed at the beginning of a period, then for the following periods, the optimal allocation quantity z^* satisfies:

(1) if $(\alpha c - r + b) + (r + p + h - \alpha c)F_2(y^*) < 0$, then $z^* = 0$;

(2) if $(\alpha c - r + b) - (p + h + b)F_1(y^*) > 0$, then $z^* = y^*$;

(3) if $(\alpha c - r + b) + (r + p + h - \alpha c)F_2(y^*) \geqslant 0$ and $(\alpha c - r + b) - (p + h + b)F_1(y^*) \leqslant 0$ then z^* is determined by:

$$(r + p + h - \alpha c)F_2\left(y^* - z^*\right) - (p + h + b)F_1\left(z^*\right) = r - \alpha c - b. \tag{4}$$

Discussion

A special case of the model occurs when the store only sells products to the customers from one channel, either online or offline. This problem can be formulated by simply modifying the constraint $0 \leqslant z \leqslant y$ in Equation (3) to $z = y$ if only offline sales are considered or $z = 0$ if only online sales are considered. It is straightforward to see that concavity is maintained and that a base-stock ordering policy is optimal.

Our results can also be extended to the situation where the firm commits to providing a minimum level of service for offline demand, β_1, which is defined in terms of an in-stock probability. When z units are allocated to offline demand, the service commitment can be characterized as $F_1(z) \geqslant \beta_1$ or equivalently $z \geqslant F_1^{-1}(\beta_1)$. We only need to add this constraint to Equation (1). Since $z \geqslant F_1^{-1}(\beta_1)$ is a sublattice, adding this constraint does not affect the preservation of the $L^{\natural}$-concavity. Therefore, the structural properties and the optimal policy we obtained still hold. Regardless, in this case, the optimal base-stock level must be large enough to ensure that the service requirement is met, that is, the optimal base-stock level is modified to $\max\{F_1^{-1}(\beta_1), y^*\}$.

Numerical study

In this section, we provide two numerical examples to illustrate the optimal allocation policy and conduct a sensitivity analysis to gain insight into how the optimal policy changes with the relevant parameters. The two examples are based on a basic apparel product and a home appliance product. We test $T = 10$ and assume $\alpha = 1$. Offline and online demands are Poisson distributed with a mean of 5 in each period. The other parameters are given in Table I.

We first test how the change in the franchisee's revenue share r affects the optimal policy. In the apparel example, we fix other parameters and increase r from 0 to 12. Our numerical study confirms the analytical results obtained in P2; that is, there is a base-stock

level for ordering, and the optimal allocation quantity for offline orders is non-decreasing in the inventory on hand. For all $r \in [0, 12]$, our study has the same optimal order-up-to level of 19. Figure 1 illustrates the optimal allocation policy for $r = 0$, $r = 6$ and $r = 12$.

It is therefore confirmed that z^* is non-decreasing in the inventory on hand. We find that z^* is insensitive to r for inventory levels between 19 and 30, and when r increases from 0 to 12, z^* remains at the same value or only decreases slightly. It is intuitive that when r is large, it is more desirable to allocate a larger portion of the inventory to online customers. The insensitivity reveals that the franchisor's sharing of revenues with their franchisees does not sufficiently motivate the latter to allocate their in-store inventory to either an online or an offline channel. This result therefore also reveals that a franchisor in an omnichannel retailing context needs to leverage legitimate power to solve channel conflicts by stipulating the associated terms and measures in the franchise contracts and requiring franchisees to reserve a certain percentage of their inventory for the franchisor's online orders.

The optimal allocation results of the home appliance product example show a similar pattern (Figure 2 depicts the results of $r = 0$, $r = 35$ and $r = 70$).

We further use Equation (4) to understand why z^* is sensitive to r for most of our numerical examples. To simplify our analysis, we assume that both demands are uniformly distributed between 1 and 10 and focus on the cases where the inventory allocated to online or offline does not exceed 10. In these situations, Equation (4) becomes:

$$(r+p+h-\alpha c)(y^*-z)-(p+h+b)z = 10(r-\alpha c-b),$$

or:

$$z = \frac{(y^*-10)r+10(\alpha c+b)+y^*(p+h-\alpha c)}{r+2p+2h+b-\alpha c}.$$

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Product type	Online and offline price: p ($)	Unit purchase cost: c	Holding cost: h	Backordering cost: b ($)	Franchisee's revenue share: r
Apparel	42	$30 (gross margin 40%)	$0.875 (holding cost 2.08%)	7	[$0, $12]
Home appliance products	270	$200 (gross margin 35%)	$5.625 (holding cost 2.08%)	10	[$0, $70]

Table I.
List of base parameters used for the numerical analysis
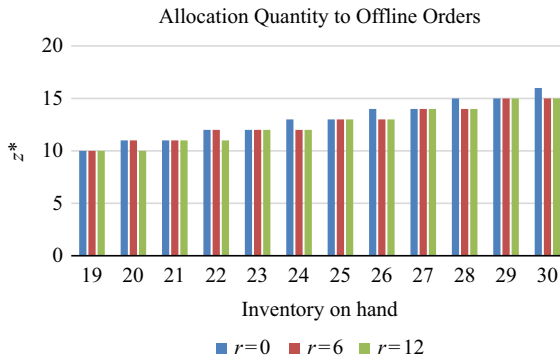
Figure 1.
Apparel product example

Notes: Optimal allocation quantity to offline orders for $r=0$, $r=6$ and $r=12$
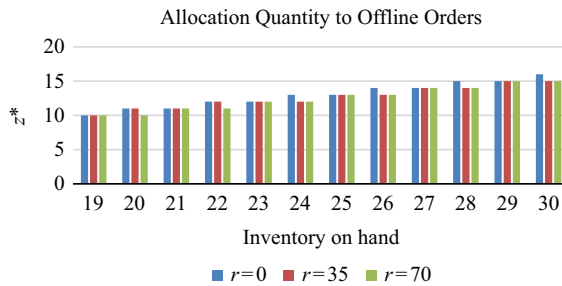
Notes: Optimal allocation quantity to offline orders for $r=0$,
$r=35$ and $r=70$

Figure 2.
Home appliance
product example

We use the base parameters of the apparel example and conduct three groups of experiments. In each group, we change one parameter to an extremely large value and fix the other parameter values. The optimal y^* and z and the relevant z formulas are given in Table II. From the z formulas, it is clear that although z is a function of r, changing r does not cause a significant change in z. This explains why franchisor's sharing of revenues with their franchisees does not sufficiently motivate the latter ones to allocate their in-store inventory to either an online or an offline channel.

Next, we test how the optimal policy changes with h and b. First, we fix the other parameter values in the apparel example and test $h = 0.735, 0.805, 0.875, 0.945$ and 1.025, which represent the annual holding cost rates of 21, 23, 25, 27 and 29 percent, respectively. Our results show that the optimal policy is insensitive to the holding cost rate. The optimal order-up-to level remains at 19 and decreases to 18 when h increases to 0.875. The optimal allocation quantity is the same for all five h parameters. Second, we fix $h = 0.875$ and other parameters and test $b = 3, 5, 7, 9$ and 11. Again, our results show that the optimal ordering and allocation policy is insensitive to the backordering cost. We obtain exactly the same order-up-to value and allocation quantities for all five b values. Third, we fix other parameters and test $p = 38, 40, 42, 44$ and 46. The optimal order-up-to level increases from 18 to 19 when we increase p from 40 to 42. The optimal allocation quantities are almost the same for all p values. The results for the home appliance product are rather similar; we therefore omit them.

r	y^*	z	z formula
$h = 30$			
0	8	6	$z = -2 + (948)/(r+121)$
6	9	6	$z = -1 + (669)/(r+121)$
12	10	6	$z = 790 + (790)/(r+121)$
$b = 20$			
0	18	9	$z = (8 + 125.75)/(r+175.75)$
6	18	9	
12	18	9	
$p = 80$			
0	18	9	$z = (8 + 175.75)/(r+138.75)$
20	18	9	
40	18	9	

Table II.
Numerical study using
the base parameters
of the
apparel example

The fact that the optimal policy is insensitive to the parameters in our examples indicates that, within a reasonable range, the estimated errors of these parameters are tolerable. A very high service level may cause insensitivity due to a very high underage cost compared to the low overage cost (holding cost). In the apparel product example, with a mean of 5 for each demand, the optimal order-up-to level is 18 or 19, which ensures a very low shortage probability. Changing the parameters within a limited range does not change the critical ratio (underage cost/(underage cost + overage cost)) much. The optimal policy therefore remains quite stable when we change the parameters in our examples.

Conclusion

Increasing retailers use their in-store inventory to serve customers who buy online but pick-up the product in-store in order to provide to the customers with seamless shopping experience (Verhoef *et al.*, 2015). This store fulfillment strategy allows retailers to gain important advantages. However, it may produce also channel conflicts, especially for retailers in franchise networks. To reduce the tensions between franchisor and franchisee, this study suggests considering the optimization of an omnichannel retailer's inventory management from its franchisees' perspective (in-store inventory management), instead of from its own perspective (firm-level inventory management). This research is triggered by several managerial studies (ENC, 2016; Hobkirk, 2016) which suggest reserving a certain percentage of the in-store inventory for online orders as a good store inventory allocation practice, but have clarified not yet how store managers can decide on the amount of inventory to replenish and the percentage to reserve for online orders.

To gain insights into these issues, this study develops a finite horizon, periodic review inventory model and examines an optimal and dynamic replenishment and allocation policy. The analysis uncovers a fundamental system structural property – L^{e}-concavity. The research shows that, due to this property, the optimal replenishment policy is a base-stock policy because the base-stock level is independent of the initial inventory at hand and the optimal allocation level is non-decreasing relative to the base-stock level. This study further shows that a myopic policy is optimal, which circumvents the curse of dimensionality for the multi-period inventory model.

This paper contributes to the literature on omnichannel retailers' inventory management by investigating firms' optimal in-store inventory ordering and allocation policy in franchising networks. Although previous research (e.g. Cheng *et al.*, 2016; Xu *et al.*, 2017) describes optimizing firm-level inventory ordering and allocation through online and offline orders, it has not considered the special organizational characteristics of omnichannel retailers in franchise networks. To address this knowledge gap, this study investigates in-store rather than firm-level inventory management to reduce the tensions between franchisor and franchisee, develops a finite horizon, periodic review inventory model, and examines an optimal and dynamic in-store replenishment and allocation policy.

The implication for retailers in franchise networks is that they should, in general, require their franchisees to allocate their inventories to either an online or offline channel by stipulating the associated terms and measures in their franchise contracts. Our analytical model shows that the implementation of such practice through leveraging legitimate powers efficiently helps firms to optimize in-store inventory management whilst addressing the problem of channel conflicts. Furthermore, our findings help store managers to optimize their decisions about inventory levels for both the online and the offline demand and the percentage of inventory to reserve for online orders.

There are several possible directions for future research. First, it could be interesting to generalize our model to more realistic cases by relaxing some of our assumptions. For

example, will our theoretical results hold with respect to a system with a replenishment lead time? What happens if there are multiple channels? What if the price promotions in the online and the offline channels differ? What if there is a fixed setup ordering cost? Second, we will conduct an empirical study and use real data to test the efficiency of our theoretical results.

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Appendix

Proof of *P1*

We prove by induction. It is obvious that $f_{T+1}(x)$ is concave. Assume $f_{t+1}(x)$ is concave. Following Zipkin (2008) (proof of Theorem 4), Equation (3) can be reformulated as the following optimization problem: after demand is realized, decide how much demand of D_2 to fill. Let a be this decision. Suppose d_1 and d_2 are realized demands. Since there is no benefit to preserve inventory before satisfying all online demand, obviously the optimal decision of a is to fill online demand as much as possible, i.e., $a = \min\{d_2, y{-}z\}$. Accordingly, let $w = (y{-}z{-}d_2)^+$ be the remaining inventory for the online channel. The problem can be written as follows:

$$J_t(y,z) = E_{d_1,d_2}\left[\bar{J}_t(y,z|d_1,d_2)\right],$$

where:

$$\bar{J}_t(y,z|d_1,d_2) = \max_{a,w}\left\{-rz-(r+p+h)w-(p+h)(z-d_1)^+ -b(d_1-z)^+ +\alpha f_{t+1}(w+z-d_1)\right\}$$

$$s.t. w \geqslant 0,$$
$$a+w = y-z,$$
$$0 \leqslant a \leqslant d_2. \tag{A1}$$

The objective function is jointly concave in (y, z) and (a, w) for given (d_1, d_2). Hence, $J_t(y, z)$ and $G_t(y)$ are also concave. Because the constraint $y \geqslant x$ in Equation (2) infers a convex set, $f_t(x)$ is also concave.

To show the $L^{\natural}$-concavity, let $\tilde{w} = w{+}z$. Equation (A1) is equivalent to:

$$\bar{J}_t(y,z|d_1,d_2) = \max_{\tilde{w}}\left\{-rz-(r+p+h)(\tilde{w}-z)-(p+h)(z-d_1)^+\right.$$

$$\left. -b(d_1-z)^+ +\alpha f_{t+1}(\tilde{w}-d_1)\right\}$$

$$s.t. \tilde{w}-z \geqslant 0,$$
$$0 \leqslant y-\tilde{w} \leqslant d_2. \tag{A2}$$

The set of $(\tilde{w},y,z)$ is a sublattice (Topkis, 1998). In addition, it is easy to check that the value function before the maximization is supermodular in $(\tilde{w},y,z)$. According to Topkis's (1998) Theorem 2.7.6 $\bar{J}_t(y,z|d_1,d_2)$ is $L^{\natural}$-concave in (y, z) for any given (d_1, d_2). Since $L^{\natural}$-concavity is preserved under the expectation, $J_t(y, z)$ is $L^{\natural}$-concave in (y, z). ∎

Proof of *P2*

(1) According to *P1*, $G_t(x)$ is concave. In view of Equation (2), the value function before the ordering optimization is concave. Therefore, there is a point, called the base-stock level, at which the value function reaches its maximum. Below this level, the value function increases and ordering leads to more profit, while above this level, the value function decreases and ordering more will decrease profit. It is therefore optimal to order up to this base-stock level whenever the inventory is less than this and not to order when the inventory exceeds it.

(2) From the concavity and supermodularity of $J_t(y, z)$, it follows that the optimal allocation quantity $z^*(y)$ is non-decreasing in y (Topkis, 1998, Theorem 2.8.1). In addition, the $L^{\natural}$-concavity of $J_t(y, z)$ implies that $z^*(y+\varepsilon) \leqslant z^*(y)+\varepsilon$ for any $\varepsilon \geqslant 0$ (Zipkin, 2008). ∎

Proof of *P3*

Let Ω be a collection of value functions such that for any $\varphi(x) \in \Omega$ and for any state x, there exists a constant B such that $\varphi(x) \leqslant B$, and $\varphi(x)=B$ if $x \leqslant y^*$. Obviously, by setting $B = 0$, then $\bar{f}_{T+1}(x) \in \Omega$.

To conduct the induction, we assume that $\overline{f}_{t+1}(x) \in \Omega$ with a constant B_{t+1}. In period t:

$$\overline{f}_t(x) = \max_{y \geqslant x}\left\{(r+p-c)y + \max_{0 \leqslant z \leqslant y} E\big[(\alpha c-r)z-(r+p+h-\alpha c)(y-z-D_2)^+ \right.$$

$$\left. -(p+h)(z-D_1)^+ -b(D_1-z)^+ +\alpha\overline{f}_{t+1}\big((y-z-D_2)^+ +z-D_1)\big]\right\} - \alpha c\mu_1$$

$$\leqslant \max_{y \geqslant x}\left\{(r+p-c)y + \max_{0 \leqslant z \leqslant y} E\big[(\alpha c-r)z-(r+p+h-\alpha c)(y-z-D_2)^+ \right.$$

$$\left. -(p+h)(z-D_1)^+ -b(D_1-z)^+\big]\right\} + \alpha B_{t+1} - \alpha c\mu_1$$

$$= \max_{y \geqslant x}\big\{(r+p-c)y+\theta(y)\big\} - \alpha c\mu_1 + \alpha B_{t+1}$$

$$= \pi(x) + \alpha B_{t+1}$$

$$\leqslant \pi(y^*) + \alpha B_{t+1}$$

$$= B_t,$$

where $B_t = \pi(y^*) + \alpha B_{t+1}$. Moreover, it is easy to check that state $(y-z-D_2)^+ +z-D_1$ is consistent. Thus, when $x \leqslant y^*$, the inequality is in fact an equality. Therefore, $\overline{f}_t(x) \in \Omega$ and the induction for the theorem is completed. ∎

Proof of Corollary 1
Consider the optimal allocation problem:

$$\theta(y) = \max_{0 \leqslant z \leqslant y} E\big[(\alpha c-r)z-(r+p+h-\alpha c)(y-z-D_2)^+ -(p+h)(z-D_1)^+ -b(D_1-z)^+\big].$$

For given y, let:

$$\eta(z) = E\big[(\alpha c-r)z-(r+p+h-\alpha c)(y-z-D_2)^+ -(p+h)(z-D_1)^+ -b(D_1-z)^+\big]$$

$$= (\alpha c-r)z-(r+p+h-\alpha c)\int_0^{y-z}(y-z-\xi)\varphi_2(\xi)d\xi-(p+h)\int_0^z(z-\xi)\varphi_1(\xi)d\xi$$

$$-b\int_z^\infty(\xi-z)\varphi_1(\xi)d\xi,$$

where $\varphi_1(\cdot)$ and $\varphi_2(\cdot)$ are density functions of D_1 and D_2, respectively. Using the Leibniz rule, we have:

$$\frac{d\eta(z)}{dz} = (\alpha c-r)+(r+p+h-\alpha c)\int_0^{y-z}\varphi_2(\xi)d\xi$$

$$-(p+h)\int_0^z\varphi_1(\xi)d\xi+b\int_z^\infty\varphi_1(\xi)d\xi$$

$$= (\alpha c-r)+(r+p+h-\alpha c)F_2(y-z)-(p+h)F_1(z)+b(1-F_1(z))$$

$$= (\alpha c-r+b)+(r+p+h-\alpha c)F_2(y-z)-(p+h+b)F_1(z).$$

Since $(d^2\eta(z))/(dz^2) = -(r+p+h-\alpha c)\varphi_2(y-z)-(p+h+b)\varphi_1(z) \leqslant 0$, the optimal allocation quantity z^* (for given y) can be determined as follows:

(1) if $d\eta(z)/dz\big|_{z=0} < 0$, then allocating any inventory to the offline channel reduces the expected profit, i.e., $z^* = 0$;

(2) if $d\eta(z)/dz\big|_{z=y} > 0$, then it is optimal to allocate as much as possible to the offline channel, i.e., $z^* = y$;

(3) if $(d\eta(z))/(dz)\big|_{z=0} \geqslant 0$ and $(d\eta(z))/(dz)\big|_{z=y} \leqslant 0$ and, then the optimal allocation decision z^* ($0 \leqslant z^* \leqslant y$) satisfies the first-order condition:

$$(\alpha c - r + b) + (r + p + h - \alpha c)F_2(y-z) - (p+h+b)F_1(z) = 0. \qquad \blacksquare$$

This completes the proof.
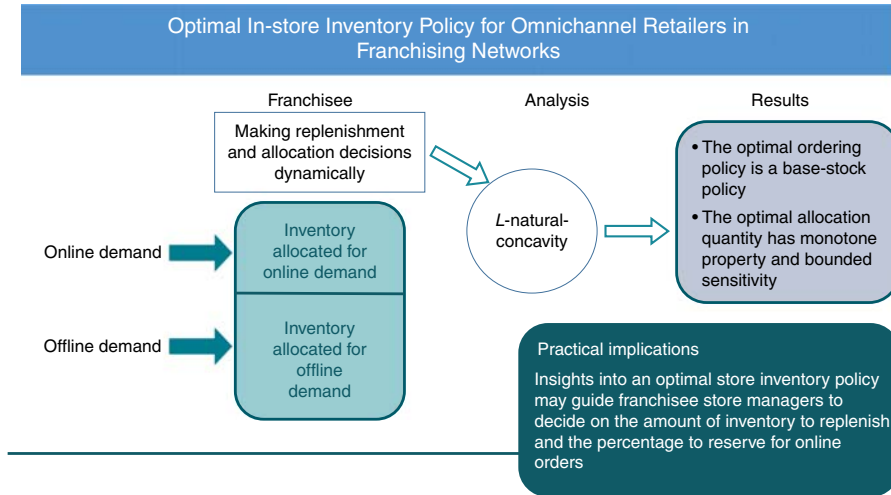
Figure A1.
Visual abstract

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